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ESTIMATING DIFFUSION CHARACTERISTICS OF TIDAL WATERS

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ESTIMATING DIFFUSION CHARACTERISTICS OF TIDAL WATERS

I. INTRODUCTION

The mathematical models available[1,2] for the analyses of pollution and other problems involving mixing in tidal waters require a knowledge of the turbulent diffusion characteristics of the system being studied. These characteristics may be evaluated in an estuary by using external tracers such as dyes or internal tracers which are already present, as in the case of salinity intrusion or a recognizable continuous pollutant discharge. Similar procedures may be applied to a verified hydraulic model of the system, where such a model exists. However, these methods can be expensive and time consuming.

In cases where preliminary calculations are being made, and general order of magnitude answers will suffice, it is most useful to be able to make satisfactory estimates of diffusion characteristics using readily available hydrographic information. Furthermore, in planning dye tracer studies, it is helpful to have some reasonable estimate of diffusion properties, to permit better design of field activities.

Several methods which have been proposed for estimating a diffusion coefficient for tidal waters are described below. In each case, it is the turbulent analog of the Fickian coefficient of molecular diffusion which is used. This has been variously

referred to in the literature as the coefficient of turbulent, eddy, or longitudinal (for the one-dimensional case) dispersion, diffusion, or mixing.

Four-Thirds Law

It has been almost universally observed that the magnitude of the turbulent diffusion coefficient increases greatly with the size of the area, or volume, containing the diffusing substance. This has led to the formulation of a "four-thirds law" wherein the diffusion coefficient is proportional to the four-thirds power of the scale of the diffusion phenonema, L. This is quoted by Bowden[3] as:

$$D_{cm^2/sec} = 0.0246L^{4/3} \quad \text{---------- Eq (1)}$$

Where L is in cm.

For the one-dimensional estuary, the characteristic scale (L) may be considered as the length of a tidal excursion, or the maximum longitudinal distance a particle of water would travel during a flooding or ebbing tide. This length would be given by:

$$L = \overline{V}t \quad \text{---------- Eq (2)}$$

L = length

$\overline{V}$ = average tidal velocity

t = period of time between slack waters (generally estimated at 6.2 hours for the Potomac Estuary)

If a sinusoidally varying tidal current is assumed:

$$\bar{V} = \frac{2}{\pi} V_{max} \text{ ---------- Eq (3)}$$

From Eq (2)

$$L_{cm} = \frac{2}{\pi} V_{max} t \text{ ---------- Eq (4)}$$

Where: V_{max} is in cm/sec

t is in sec

$$L_{cm} = \frac{2}{\pi} (6.2 \text{ hrs.}) (3600 \text{ sec/hr.}) V_{max}$$

$$L_{cm} = 14{,}200\ V_{max}$$

By substituting into Equation (1):

$$D_{cm^2/sec} = 0.0246\ [14{,}200\ V_{max}]^{4/3}$$

$$= 0.0246\ [333{,}000]\ V_{max}^{4/3}$$

$$= 8.21 \times 10^3\ V_{max}^{4/3} \text{ with } V_{max} \text{ in cm/sec -- Eq (5)}$$

Equation (5) may be restated using units of knots for V_{max}. This form of the equation may be more convenient, as values for V_{max} in knots may be obtained from published tables for many estuarial locations.[4]

$$D_{cm^2/sec} = 1.56 \times 10^6\ V_{max}^{4/3} \text{ ---------- Eq (6)}$$

Where V_{max} is in knots

Other forms of Equation (6) are as follows:

$$D_{mi^2/day} = 5.2\ V_{max}^{4/3} \text{ ---------- Eq (6a)}$$

Where V_{max} is in knots

$$D_{ft^2/sec} = 1.68 \times 10^3\ V_{max}^{4/3} \text{ ---------- Eq (6b)}$$

Where V_{max} is in knots

This procedure was carried out for the Potomac River below Washington, D. C., and the results are plotted in Figure 1.

Random Process Analogy

The process of tidal mixing and dispersion has been described by Diachishin[5] as a random walk type of process. Using probability theory, he has developed equations which resemble solutions of the classical diffusion equation. For this similarity to be complete, it was concluded that the diffusion coefficient must be described as follows:

$$D = \frac{nL^2}{2} \qquad \text{Eq (7)}$$

where L is the length of each random step and n is the frequency of steps (one per tidal cycle or 1/12.4 per hour). Applying a procedure similar to that above for defining the scale length, an average step length is used which is equivalent to the tidal excursion length, as given in Equation (4). Combining these expressions results in the relationship:

$$D_{cm^2/sec} = \frac{1}{2}\left(\frac{1}{12.4\ hr}\right)\left(\frac{1}{3{,}600\ \frac{sec}{hr}}\right)(14{,}200\ V_{max})^2 \qquad \text{Eq (8)}$$

$$D_{cm^2/sec} = 2.26 \times 10^3\ V_{max}^2$$

with V_{max} in cm/sec

or in the more convenient form

$$D_{cm^2/sec} = 6.2 \times 10^6\ V_{max}^2 \qquad \text{Eq (8a)}$$

where V_{max} is in knots

$$D_{mi^2/day} = 20.7\ V_{max}^2 \qquad \text{Eq (8b)}$$

where V_{max} is in knots

$$D_{ft^2/sec} = 6.68 \times 10^3\ V_{max}^2 \qquad \text{Eq (8c)}$$

where V_{max} is in knots

Coefficients for the Potomac Estuary calculated using this equation are also shown in Figure 1.

Turbulent Pipe Flow Analogy

Taking Taylor's expression for the longitudinal dispersion coefficient for turbulent flow in pipes as a starting point, Harlemann[6] has developed an analogous expression for tidal waters. In this development, it has been assumed that the required steady, turbulent flow velocity may be approximated by the time average of the tidal velocity over a half tidal cycle. In addition, a sinusoidal velocity variation has been assumed. The resulting expression is:

$$D = 77n \left(\frac{2}{\pi}\right) V_{max} R^{5/6} \quad \text{---------- Eq (9)}$$

where R (ft) is the hydraulic radius for which the mean depth is a satisfactory approximation, n is Manning's roughness coefficient, and V_{max}(fps) is the maximum tidal velocity. For the Delaware Estuary, Harlemann estimated n to be 0.025. It is also recommended, however, that the final result be multiplied by 1.5, to account for bends and section changes, so an effective n of 0.038 has been used in the following reduced equation:

$$D_{cm^2/sec} = 2.92 \times 10^3 V_{max} R^{5/6} \quad \text{---------- Eq (10)}$$

with V_{max} in knots and R in feet.

In a similar manner, but employing a slightly different set of assumptions, Bowden[7] derived the expression:

$$D_{cm^2/sec} = 235\ V_{max} R \qquad \text{Eq (11)}$$

With V_{max} and R defined as above.

Results obtained using these formulas on the Potomac Estuary are shown in Figure 1.

II. DISCUSSION

The first two methods described (Equations 6 and 8) produce somewhat similar results (Figure 1). These coefficients, in turn, differ from those obtained by the turbulent pipe flow analogy formulas by two to three orders of magnitude.

In order to provide a basis for comparison of the absolute values obtained by these formulas, the coefficient of diffusion was also calculated from the salinity distribution in this portion of the estuary. The solution to the classical diffusion equation may be expressed in forward finite difference form as:[8]

$$D = \frac{Q\, S_x\, 2\Delta x}{A_x\, (S_{x+\Delta x} - S_{x-\Delta x})} \qquad \text{Eq (12)}$$

Knowledge of the longitudinal salinity distribution (S), net advective flow (Q) and cross-sectional area (A) in the estuary will permit solution for D. This was carried out for relatively steady flow conditions near 2000 cfs in the Potomac Estuary, using chloride data provided by the District of Columbia Department of Sanitary Engineering.

Coefficients obtained by solution of Equation (12) are closely approximated by the "four-thirds" model and are less than those obtained by the random process formula by a factor of about three. The turbulent pipe flow analogy equations, however, produce coefficients for the Potomac Estuary that are lower by multiples of 100 to 1000 than the observed values. Coefficients calculated for the Patuxent Estuary, using the above equations, showed differences of the same order of magnitude as those calculated for the Potomac.

This large discrepancy was also noted by the respective sources from which these equations were obtained. Harlemann stated that the low values obtained by Equation (10) are valid for the fresh water portions of tidal streams. The much higher diffusion coefficients observed in the brackish portions of tidal waters he attributes to density effects. The longitudinal salinity gradient existing in estuaries is said to cause a large scale gravitational circulation with the more saline waters moving upstream along the bottom and downstream near the water surface. The over-all effect of this circulation, referred to by Bowden as a density current, is to cause a large increase in the observed effective diffusion coefficient. Harlemann has demonstrated the validity of Equation (10) in hydraulic models, but, apparently, not as yet in prototype estuaries.

III. CONCLUSIONS

Both the "four-thirds law" and random process analogy formulas yield effective diffusion coefficients in the proper order of magnitude for the brackish portion of the Potomac Estuary. Either of these formulas should provide useful approximations for other estuaries where good information on salinity variations is not readily available. These formulas should be used with caution in the fresh water portion of tidal rivers, in view of the possible important influence of salinity gradient induced density currents on diffusion. Prototype tracer studies in such locations appear to be necessary to provide reliable estimates of the appropriate diffusion coefficient. Accumulation of experience gained by such studies will permit evaluation of the validity of Equations (10) and (11) for predictive purposes.

BIBLIOGRAPHY

1. Thomann, R. V., "Mathematical Model for Dissolved Oxygen," JSED, ASCE, 89, No. SA5, (October 1963)

2. O'Connor, D. J., "Oxygen Balance of an Estuary," JSED, ASCE, 86, No. SA3, (May 1960)

3. Bowden, K. F., "The Sea, Volume One, Section VI, Turbulence," p. 819, Interscience Publishers, New York (1962)

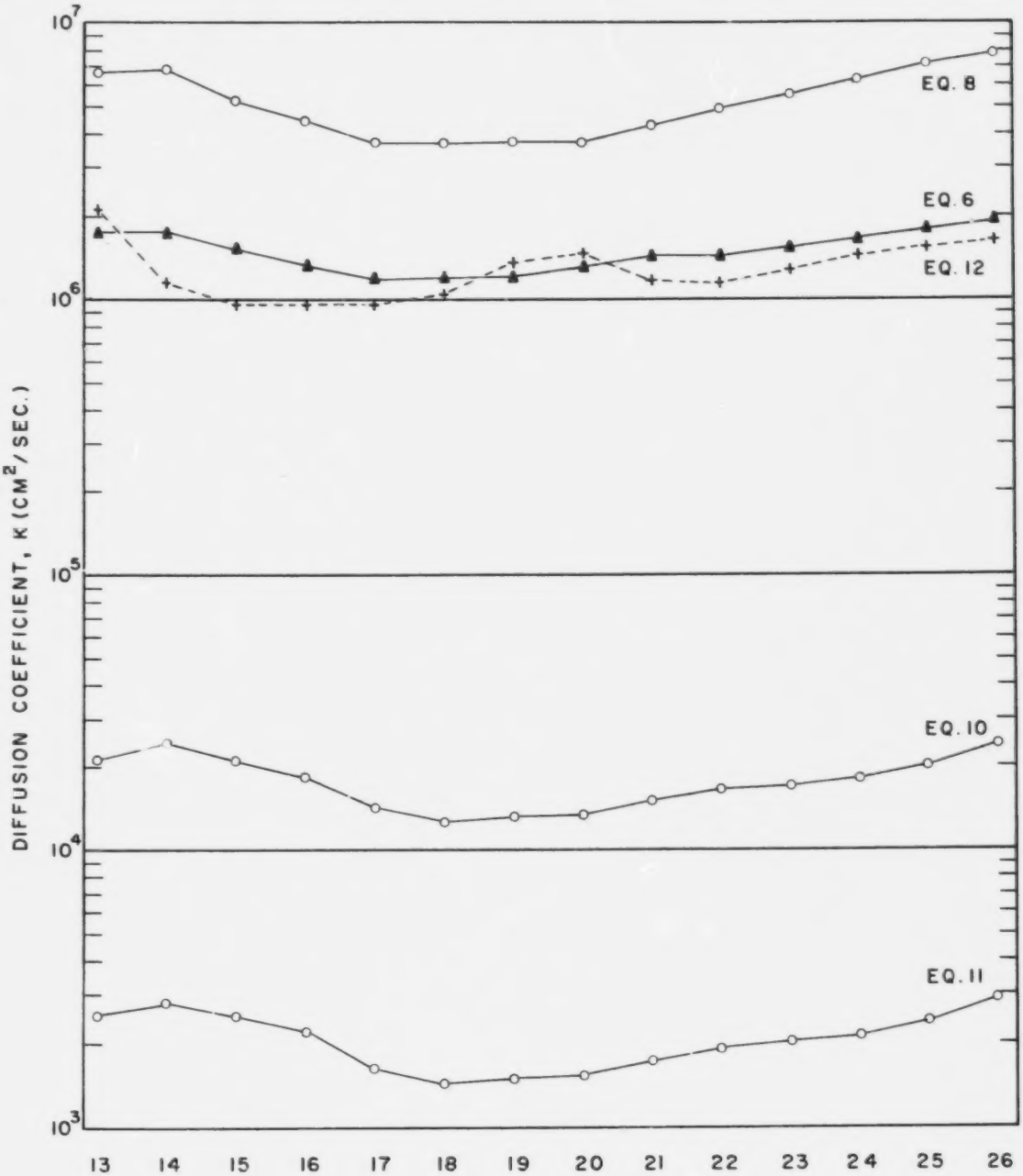
4. "Tidal Current Tables, Atlantic Coast of North America," U. S. Coast and Geodetic Survey, Department of Commerce, U. S. Government Printing Office (1965)

5. Diachishin, A. N., "Waste Disposal in Tidal Waters," JSED, ASCE, 89, No. SA4, (August 1963)

6. Harlemann, D. R. F., "The Significance of Longitudinal Dispersion in the Analysis of Pollution in Estuaries," Presented at Second International Conference on Water Pollution Research, Tokyo, Japan, (August 1963)

7. Bowden, K. F., "The Mixing Processes in a Tidal Estuary," Advances in Water Pollution Research, Vol. 3, Pergamon Press, The MacMillan Co., New York (1964)

8. Stommel, H., "Computation of Pollution in a Vertically Mixed Estuary," SIW, 25, 9, (September 1953)



DIFFUSION CHARACTERISTICS, POTOMAC RIVER ESTUARY

FIGURE 1